THE ELLIPTIC FUNCTIONS.

The Applications of Elliptic Functions. By ALFRED GEORGE GREENHILL, M.A., F.R.S., Professor of Mathematics in the Artillery College, Woolwich. London and New York, Macmillan & Co., 1892. 8vo, pp. xi + 357.

IN recent years there has been a tendency to regard pure and applied mathematics as subjects to be kept clearly distinct. The increasing necessity for close specialization, occasioned by the enormous yearly output of memoir literature, is no doubt largely responsible for this unfortunate condition of things, but there are other elements which enter into the question. To a large class of pure mathematicians the lack of logical rigor in the current proofs of physical theorems is abhorrent; to them it seems that a treatise on any subject in pure mathematics should use methods peculiar to that subject, and that no appeal should be made to extraneous physical considerations. The applied mathematician, on the other hand, is too apt to speak scornfully of the works of his rival. It is indisputable that the problems which confront the physicist of to-day require for their adequate treatment the most refined methods in pure analysis. It is, we believe, equally certain that many portions of modern mathematics must remain, so to speak, in the air, until they are connected with the physical considerations from which they originated. Professor Greenhill's sympathies are largely with the practical man as against the theorist; the first words of his introduction are a quotation from Fourier in which occur the words "les questions vagues et les calculs sans issue." At the same time he has earned for himself a high reputation by his researches on the complex multiplication of elliptic functions, a subject without any very immediate applications. Professor Greenhill has rendered an important service to the mathematical public by showing how easily elliptic functions lend themselves to the solution of dynamical and other problems; an especially valuable feature of his treatise is the free use which he has made of the Weierstrassian functions.

In the first chapter there is a detailed treatment of pendulum motion, and in the course of this discussion several of the fundamental properties of $\mathrm{sn}u$, $\mathrm{cn}u$, $\mathrm{dn}u$ are established. It is shown that they have a real period and that they may degenerate into circular and hyperbolic functions. There are obvious advantages in beginning the subject with a concrete example; but we doubt whether this is the way "best calculated to define the elliptic functions, and to give the student an idea of their nature and importance." The all-important