It is to be distinctly observed that in this process we do not require the functions fx and $\sum\limits_0^{n-1} A_r\phi_r x$ to have a contact of the $(n-1)$th order at $x = a$ in order that we may equate their first $n-1$ derivatives when $x = a$. What we require is merely that the functions fx and $\varphi_r x$ $(r = 1 \ldots n-1)$ shall each have a determinate derivative at $x = a$, up to the $(n-1)$th operation. Of course, if fx and $\sum\limits_0^{n-1} A_r\phi_r x$ have an $(n-1)$th contact at $x = a$, then our value for R holds true as well; but it is not dependent on such a relation: it simply includes it.

If now the successive functions $\phi_r x$ $(r = 1 \;.\;.\;.\; n)$ may be formed in succession indefinitely according to a given law so that we may make r in $\phi_r x$ as great as we choose, then if it can be shown that R has for its limit zero, as r becomes infinite and at the same time the A's have limiting values such that $\sum\limits^{\infty} A_r\phi_r x$ is a converging series, then we may write

$$fx = A_0 + A_1\phi_1 x + A_2\phi_2 x + \;.\;.\;.\; \text{ad. inf.}$$

The value of R has been shown to be

$$\frac{\begin{vmatrix} 1, & \phi_1 x & \ldots & \phi_n x \\ 1, & \phi_1 a & \ldots & \phi_n a \\ 0, & \phi_1' a & \ldots & \phi_n' a \\ \cdot & \cdot & \cdot & \cdot \\ 0, & \phi_1^{n-1} a & \ldots & \phi_n^{n-1} a \end{vmatrix} \left(\dfrac{d}{dx}\right)^n_{x=u} \begin{vmatrix} fx, & 1, & \phi_1 x & \ldots & \phi_{n-1} x \\ fa, & 1, & \phi_1 a & \ldots & \phi_{n-1} a \\ f'a, & 0, & \phi_1' a & \ldots & \phi_{n-1}' a \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f^{n-1}a, & 0, & \phi_1^{n-1} a & \ldots & \phi_{n-1}^{n-1} a \end{vmatrix}}{\begin{vmatrix} \phi_1' a & \ldots & \phi_{n-1}' a \\ \cdot & \cdot & \cdot \\ \phi_1^{n-1} a & \ldots & \phi_{n-1}^{n-1} a \end{vmatrix} \left(\dfrac{d}{dx}\right)^n_{x=u} \begin{vmatrix} 1, & \phi_1 x & \ldots & \phi_n x \\ 1, & \phi_1 a & \ldots & \phi_n a \\ 0, & \phi_1' a & \ldots & \phi_n' a \\ \cdot & \cdot & \cdot & \cdot \\ 0, & \phi_1^{n-1} a & \ldots & \phi_n^{n-1} a \end{vmatrix}} \tag{11}$$

in which u is some unknown value of x lying between x and a.

ON THE EARLY HISTORY OF THE NON-EUCLIDIAN GEOMETRY.

BY EMORY McCLINTOCK, LL.D.

It has until recently been supposed that the earliest work on non-euclidian geometry was Lobatschewsky's.* A much earlier production (1733) has been brought into notice by

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* See BULLETIN of November, 1892, vol. II, No. 2, "On the Non-Euclidian Geometry."