

A BIT OF MATHEMATICAL HISTORY.

BY DR. MAXIME BÔCHER.

IN the transactions of the Academy of St. Petersburg for the year 1764 is a memoir by Leonhard Euler entitled : *De Motu Vibratorio Tympanorum*, which is remarkable in several respects. Certainly the last part of this memoir has been lost sight of for many years by mathematicians. Whether the same can be said of the first part I do not know,* but I will venture to give a brief account of the contents of the whole paper.

During the second third of the eighteenth century a number of mathematicians, among whom Daniel Bernoulli and Euler deserve special mention, had given much attention to problems concerning the vibration of such one dimensional bodies as strings, rods and columns of air, but the paper now under consideration is the first in which a two dimensional problem of the same nature was taken up ; the question chosen being the problem of the vibration of a stretched elastic membrane. The now familiar equation of motion of the membrane :

$$\frac{1}{e^2} \frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

is first obtained, and then follows a brief discussion of the simple harmonic vibrations of a rectangular membrane, but although the square membrane is chosen as a simple case in which to give numerical results the peculiar interest of this case is not mentioned.

Failing to find a general method for the discussion of the vibrations of membranes of *any* shape, Euler next takes up the question of the vibration of a circular membrane. The first step towards the solution of this problem is the introduction of polar in place of rectangular coördinates in the equation of motion, and even this step is worthy of notice as being apparently the first case of a change of independent variables in a partial differential equation.†

* Certainly Poisson did not know of the existence of this paper when he wrote his celebrated memoir on elasticity (*Mémoires de l'Institut*, 1829) which is frequently referred to as containing the first investigation of the vibration of membranes.

† Euler himself says : *Modus autem, quo hanc æquationem elicimus, novam quandam algorithmi speciem constituit, quæ omni attentione digna videtur.*