ON PETERS'S FORMULA FOR PROBABLE ERROR.*

BY PROF. W. WOOLSEY JOHNSON.

THERE are three quantities which may be used as measures of the risk of error in an observation, each of which, according to the law of facility, bears a fixed ratio to the reciprocal of the measure of precision; namely, the probable error r, the mean error ε, and the mean absolute error η. Their values, in terms of h, namely

$$r = \frac{\rho}{h}, \qquad \varepsilon = \frac{1}{h\sqrt{2}}, \qquad \eta = \frac{1}{h\sqrt{\pi}},$$

may be called their *theoretic values*. By definition these would, in an infinite number of observations of the same precision, be respectively: the error which in order of absolute magnitude stands in the middle of the series, that whose square is the mean of the squares of the errors, and the mean of the absolute values of the errors.

The quantities similarly defined with reference to a series of n given observations may be called the *observational values* of r, ε and η. The assumption of the equality of the theoretic and observational value of either of the measures of the risk of error will assign a value to h and to each of the other measures. With respect to r, such an assumption would obviously be unsatisfactory, except when n is very large; but with respect to ε and η, whose observational values are

$$\sqrt{\frac{\Sigma e^2}{n}} \qquad \text{and} \qquad \frac{\Sigma\,[e]}{n},$$

the assumptions give the methods of determining h and r which are actually in use. Thus the ε-method gives

$$r = \rho\sqrt{2}\sqrt{\frac{\Sigma\, e^2}{n}} = .6745\sqrt{\frac{\Sigma\, e^2}{n}}\,; \quad . \quad . \quad . \; (1)$$

and the η-method gives

$$r = \rho\sqrt{\pi}\,\frac{\Sigma\,[e]}{n} = .8453\,\frac{\Sigma\,[e]}{n}. \quad . \quad . \quad . \; (2)$$

The first formula is preferred because we can prove otherwise that the corresponding value of h is the most probable

* Abstract of a Paper read before the Society, June 4, 1892.