MATHEMATICAL PROBLEMS.

Solution of Questions in the Theory of Probability and Averages. Appendix II. to *Mathematical Questions and Solutions from the Educational Times,* Vol. LV. By Professor G. B. ZERR, M.A.

THIS pamphlet of fifty-six pages contains solutions of more than forty problems in geometrical probability and mean values and of some other interesting mathematical problems. The solver was also the proposer of most of the problems. His solutions show skill and perseverance in evaluating many complicated definite multiple integrals.

Several problems relate to mean values of magnitudes determined by choosing random points with certain restrictions in a circle. For example, No. 11,153 is to find the average area of the dodecagon formed by joining twelve points taken at random in a circle, three in each quadrant. The expression found for this area is the quotient of two multiple integrals of twelve variables each. This is finally reduced to

$$\frac{2^{14} r^2}{15\pi^6}\left(\frac{\pi^2}{16}-\frac{704}{1575}\right)\left(2\pi-\frac{409}{105}\right)+\frac{2^7 r^2}{\pi^3}\left(\frac{5\pi^2}{64}-\frac{29}{45}\right)$$
$$+\frac{2^{12} r^2}{3\pi^6}\left(\frac{\pi^2}{32}-\frac{86}{315}\right)\left(\frac{7\pi}{2}-\frac{853}{105}\right)$$

Problem 11,037 is as follows :—" Two points are taken at random in the surface of a given circle. An ellipse is described on the distance between the two points as major axis. If a point be taken at random in the left-hand half of this major axis, and with this point as a centre a circle is described at random, but so as to lie wholly within the ellipse, find the average area of the ellipse described on that portion of the major axis between the right-hand extremity and the circumference of the random circle." The result obtained is

$$\frac{\pi r^2}{1280}\left(\frac{2205\pi+2012}{15\pi+17}\right).$$

This solution involves the assumption that the major axis a of an ellipse being given all possible ellipses should be included by taking the unknown minor axis as an independent variable with the limits 0 and a. All possible ellipses might with equal propriety be included in other ways ; as, by taking the eccentricity as the independent variable with the limits 0 and 1. The result would be altered by such a change.

Problem 11,130 implies an assumption of the same nature.