

## KLEIN'S MODULAR FUNCTIONS.

FELIX KLEIN, *Vorlesungen über die Theorie der elliptischen Modulfunctionen*, ausgearbeitet und vervollständigt von Dr. ROBERT FRICKE. Erster Band. Grundlegung der Theorie. Leipzig, Teubner, 1890. 8vo, pp. xix + 764.

THE mathematical public is under great obligation to Professor Klein's former pupil, Dr. Robert Fricke, for his able presentation of the theory of the modular functions. His clearness of treatment and skillful grouping of the many intricate features of the subject have rendered this theory now thoroughly accessible. Beside the work of arrangement, in itself a labor of no small magnitude, Dr. Fricke has contributed many of the intermediate steps necessary to the symmetry and completeness of the subject. His task has been performed throughout with a highly creditable degree of conscientiousness and ability.

The theory of the modular functions and the allied branches has been one of the chief series of investigations to which Professor Klein has devoted himself in the period of some twenty years over which his scientific activity now extends. It is characteristic of these investigations that they are not included as a subordinate part in any of the great mathematical theories heretofore commonly so recognized. Their distinctive tendency is in the direction of the combination and unification of the latter into a broader method of research. This idea has been developed by Klein to an extent and with an elaboration which have long since entitled it to recognition as an independent, and in the highest degree productive mathematical point of view. In the present paper some attempt is made to sketch the general outlines of the new method, so far as it concerns the modular functions, and to illustrate it more definitely by the consideration of some of the more important details.

Historically, Klein's work has developed accurately along the lines of a thoroughly predigested plan, the bolder features of which are already sharply defined in his earliest publications.\* On this ground, then, a brief semi-biographical, semi-scientific sketch of his career may properly find place here. It is to be observed that this sketch makes no pretension to completeness. It confines itself mainly on the scientific side to the development of the theory of the regular bodies and of the modular functions.

Klein's first productive activity dates from his relation to

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\* Cf. the preface to the "*Ikosaeder*," and the *Eintrittsprogramm* mentioned on the following page.