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ON $\mathfrak{so}_8$ AND THE TENSOR OPERATORS OF $\mathfrak{sl}_3$

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Physicists motivated by problems in high energy physics have for many years been groping their way toward a theory of tensor operators of simple lie algebras. Their work, calculations, insights, theorems, and guesses have appeared in the physics literature in a form not easily understood by either mathematicians or physicists [**2, 3**]. Extensive conversation with L. C. Biedenharn, a chief worker in this field, has convinced me that it is a mathematical gold mine. It has inspired the work described here.

In this announcement the tensor operators of $\mathfrak{sl}_3$ are analyzed in terms of a beautiful algebraic structure involving $\mathfrak{so}_8$, whose existence was previously unsuspected even by physicists.

Proofs will appear in [**1** and **5**].

The fundamental problem is the *explicit* decomposition of all finite dimensional tensor product representations $V \otimes W$ of a simple lie algebra $\mathfrak{g}$. For $\mathfrak{g} = \mathfrak{sl}_2$, the famous Clebsch-Gordan coefficients provide a complete solution.

We shall study the equivalent problem of decomposing all the $\mathrm{Hom}_{\mathbf{C}}(V, W)$, the spaces of "tensor operators".

A tensor operator is an irreducible subrepresentation of $\mathrm{Hom}_{\mathbf{C}}(V, W)$ where V and W are irreducible $\mathfrak{g}$-representations.

Irreducible representations of $\mathfrak{g}$ are labelled by their highest weights. To a tensor operator one can assign two weights: (1) the highest weight of the tensor operator as an irreducible $\mathfrak{g}$-representation; (2) the weight which is the difference of the highest weights of V and W, the "shift in representations" effected by the tensor operator. Tensor operators with the same highest weights but different "shift weights" map between different spaces and would therefore seem to be unrelated. But the fact that the shift weight of a tensor operator is necessarily an actual weight of the tensor operator as a representation suggests otherwise. Much work, including my own, can be described as the pursuit of the analogy between the pairs "actual weight and highest weight" and "shift weight and highest weight." The principal difficulty is that for $\mathfrak{g}$ other than $\mathfrak{sl}_2$ irreducible representations may have weights with multiplicity greater than one.

Denote by V_λ a (finite dimensional) irreducible representation of $\mathfrak{g}$ with highest weight λ.

The multiplicity of V_λ in $\mathrm{Hom}_{\mathbf{C}}(V_\alpha, V_{\alpha+\mu})$ is bounded by the multiplicity of the weight μ in V_λ, with equality of multiplicities for generic α (i.e., for α far from Weyl chamber walls).

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