BOOK REVIEWS

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Vertex operator algebras and the Monster, by Igor Frenkel, James Lepowsky, and Arne Meurman. Academic Press, New York, 1988, 502 pp., $69.95. ISBN 0-12-276065-5

The classification of all finite simple groups was achieved about ten years ago, the fruit of many years of work by scores of mathematicians and requiring thousands of journal pages for the proof. Besides the alternating groups and sixteen infinite families of groups of Lie type, there are twenty-six sporadic groups, the last and most glamorous being the *Monster* M, of order

$$|M| = 2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 11^{2}\cdot 13^{3}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71 \sim 8\cdot 10^{53}.$$

Its existence was conjectured in 1973 by Fischer and by Griess, together with a nontrivial rational representation of (minimal) degree

$$d_2 = 196883 = 47 \cdot 59 \cdot 71.$$

For a period of about eight years, the Monster, like its namesake in Loch Ness, was not known to exist, but enough observations had been made to support a considerable body of theory. In particular, Fischer, Livingstone, and Thorne had computed the character table, and Norton knew that the representation space of dimension 196883 must be an algebra of a certain kind.

On the other hand, we have modular functions, a subject which has been flourishing for nearly two centuries, and doing especially well in recent years; it has always been central to number theory and with close ties to many parts of mathematics. For example, the *elliptic modular invariant* $j(\tau)$ classifies the isomorphism classes