BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 25, Number 2, October 1991

CONVERGENCE GROUPS ARE FUCHSIAN GROUPS

DAVID GABAI

ABSTRACT. A group of homeomorphisms of the circle satisfying the "convergence property" is shown to be the restriction of a discrete group of Mobius transformations of the unit disk. This completes the proof of the Seifert fiber space conjecture and gives a new proof of the Nielson realization problem.

A Fuchsian group F is a discrete subgroup of the group of Mobius transformations on the unit disc D^2 in R^2. F restricts to a subgroup G of $\mathrm{Homeo}(S^1)$ which satisfies the following convergence property [GM]. Given a sequence of distinct elements of G, then there exists $x, y \in S^1$ and a subsequence $\{f_i\}$ such that on $S^1 - \{x, y\}$ $f_i \to y$, $f_i^{-1} \to x$ uniformly on compact sets. A group $G \subset \mathrm{Homeo}(S^1)$ with this property is called a convergence group. We announce the following result. The details can be found in [G].

Theorem 1. *G is a convergence group if and only if G is conjugate in* $\mathrm{Homeo}(S^1)$ *to the restriction of a Fuchsian group.**

A Seifert fibred space is a compact 3-manifold M which is almost an S^1 bundle over a compact surface, i.e. there exists a projection $\pi : M \to N$ such that for each $x \in N$ there exists a D^2 neighborhood of x such that $\pi^{-1}(D^2) = D^2 \times S^1$ and $\pi((r, \theta_1), (1, \theta_2)) = (r, p\theta_1 + q\theta_2)$ where $p \neq 0$ and p, q are relatively prime and depend on x and $\theta \in \mathbb{R} \bmod 2\pi$.

Corollary 2 (Seifert Fibred Space Conjecture). *Let M be a compact, orientable, irreducible (i.e. every smooth embedded S^2 bounds a* 3*-cell)* 3*-manifold with infinite π_1, then M is a Seifert fibred space if and only if $\pi_1(M)$ contains a cyclic normal subgroup.*

Received by the editors January 7, 1991 and, in revised form, April 20, 1991.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 57S25; Secondary 20H10, 57N05, 57N10.
Partially supported by NSF Grant DMS-8902343 and a Sloan Foundation Fellowship.

*Andrew Casson has also announced, using different methods, a proof of Theorem 1.