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The subgroup structure of the finite classical groups, by Peter Kleidman and Martin Liebeck. London Mathematical Society Lecture Note Series, vol. 129, Cambridge University Press, Cambridge, 1990, vii + 303 pp., $29.95. ISBN 0-521-35949-X

There are many questions to ask and answer about subgroups of the classical groups. The questions considered in *The subgroup structure of the finite classical groups* have their roots in the theory of permutation groups. The focus of the book is the central problem of modern permutation group theory: Describe the maximal subgroups of the finite simple groups.

To get some feeling for the questions the authors consider, let us imagine that we are permutation group theorists faced with a problem involving permutation groups. Recall first that each transitive permutation representation of a group G on a set X is equivalent to a representation of G on the cosets of some subgroup H by right multiplication. Indeed H is the stabilizer in G of a point of X. Further as permutation group theorists, we are used to reducing our questions to a problem about *primitive groups*: groups G preserving no nontrivial partition on X. Finally we know that G is primitive if and only if H is maximal in G.

Before 1980 these reductions probably would not have made a big dent in our problem. But about 1980 two related events occurred. First, Mike O'Nan and Len Scott independently made an important observation that is sufficiently elementary to have been made in the late 19th Century: the structure of a finite primitive permutation group is highly restricted. (cf. [Sc] and [ASc]). As a matter of fact most such groups are so restricted that in many problems the only primitive groups which cannot be easily analyzed are the *almost simple groups* G. That is G has a unique minimal normal subgroup L and L is a nonabelian simple group. Better, if we embed G in the group $\mathrm{Aut}(L)$ of automorphisms of L via conjugation and identify L with its group of inner automorphisms, then we have $L \trianglelefteq G \leq \mathrm{Aut}(L)$.

Thus in many circumstances we are reduced to the study of the maximal subgroups of the almost simple groups. Until 1980 even this would not buy us much, which is probably why Burnside did not prove the O'Nan-Scott theorem. However in 1981 the