

was astonished to learn that the Pell equation $x^2 - Dy^2 = 1$ had been solved as early as 3000 years ago in India. The fundamental solution of $x^2 - 8y^2 - 1$ is $9 - 8 \cdot 1 = 1$. For $D = 13$, the fundamental solution is $421201 - 13 \cdot 32400 = 1$. Since this solution appears in an ancient writing, an algorithm was known in prehistoric times.

In modern times, the idea of finding solutions in positive integers has been extended. For all sorts of polynomial and exponential equations, one looks for solutions in (some fixed) number field. It is believed that the only solution of $3^x - 2^y = 1$ is given by $9 - 8 = 1$, but I know no proof. If $a, b, c > 1$, the equation $a^x - b^y = c^z$ ($x, y, z > 2$) probably has only finitely many solutions. The famous Fermat equation is a special case. It is conjectured to have no solutions.

The Thue equation is $f(x, y) = k$, $k > 0$. Here $f(x, y)$ is a homogeneous polynomial in x, y with (say) integer coefficients. Shorey and Tijdeman proved that each such equation has only finitely many solutions. De Weger gives an algorithm for finding all solutions. For reasonably small values of k the algorithm is practicable with current technology.

Another problem is: solve $x + y = z^2$, where the prime divisors of xy are restricted to be among 2, 3, 5, 7. De Weger tabulates all solutions. The largest is $2^3 3^7 5^4 7 + 1 = 13^2 673^2$. Other large ones are $2^{12} + 3^3 \cdot 5 \cdot 7 = 71^2$, $2^5 3^2 5^2 7 + 1 = 449^2$. The smallest are $3 + 1 = 4$, $2 + 2 = 4$.

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Volterra integral and functional equations, by G. Gripenberg, S.-O. Londen, and O. Staffans. Cambridge University Press, Cambridge, 1990, 701 pp., \$99.50. ISBN 0-521-37289-5

The spirit of this book is captured in the following quotation from the introduction "The subject of Volterra equations in finite