

- [vN] J. von Neumann, *Allgemeine Eigenwerttheorie Hermitescher Funktionaloperatoren*, Math. Ann. **102** (1929-1930), 49–131.
- [W] G. Wittstock, *On invariant subspaces of positive transformations in spaces with indefinite metric*, Math. Ann. **172** (1967), 167–175. (German)
- [We] K. Weierstrass, *Zür Theorie der bilinearen und quadratischen Formen*, Monatsber. Akad. Wiss., Berlin, 1868, pp. 310–338.

LEIBA RODMAN

THE COLLEGE OF WILLIAM AND MARY

BULLETIN (New Series) OF THE  
 AMERICAN MATHEMATICAL SOCIETY  
 Volume 25, Number 1, July 1991  
 ©1991 American Mathematical Society  
 0273-0979/91 \$1.00 + \$.25 per page

*Characterization of Banach Spaces not containing  $\ell^1$* , by D. van Dulst. Centrum voor Wiskunde en Informatica, Amsterdam 1989, iv + 163 pp. ISBN 90-6196-366-4

## 1. INTRODUCTION

Central to Banach Space theory is the study of the classical Banach spaces  $c_0$ ,  $\ell^p$ ,  $L^p$  ( $1 \leq p \leq \infty$ ),  $C(K)$ , and of their relationship with general Banach spaces.

The space  $\ell^1$  is of special importance in the theory of general Banach spaces. This is due to a phenomenon of considerable interest, namely that many pathological properties of Banach spaces are closely related to the fact that they have subspaces close to  $\ell^1$ . This is true in the “local” theory of Banach spaces (see e.g. Pisier’s work [P] for a celebrated example) as well as in infinite dimensional theory, the subject of our present interest.

Among the “elementary” spaces  $c_0$ ,  $\ell^p$ ,  $p < \infty$ ,  $\ell^1$  is the only one that has a nonseparable dual. If a Banach space contains  $\ell^1$ , its dual is nonseparable. (For simplicity, we say that a Banach space contains  $\ell^1$  if it contains a subspace isomorphic to  $\ell^1$ .) It was conjectured for a long time that the converse holds. This conjecture was disproved in 1974 by a deep example of R. C. James, the so called James tree space JT. (This space and its variations remain of considerable interest.) Thus it came as a surprise that simple criteria allow one to decide whether or not a Banach space contains a copy of  $\ell^1$ . The main results in that direction were proven in 1974 by H. P. Rosenthal, and surely constitute one of the most beautiful achievements of Banach space theory. One of