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ON THE BASIS PROBLEM FOR SIEGEL-HILBERT MODULAR FORMS

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ABSTRACT. In this paper, we mainly announce the result: every Siegel-Hilbert cuspform of weight divisible by $4h$ and of square-free level relative to certain congruence subgroups is a linear combination of theta series.

INTRODUCTION

Theta series provides one of the two most explicit ways to construct holomorphic modular forms. The other way is by Eisenstein series. A virtue of theta series is that they are given in their Fourier expansions, with integral coefficients each of which is the number of lattice points of a certain length. One of the historic motivations to study modular forms was to study theta series. On the other hand, some modular forms, like the discriminant function $\Delta(z)$ of elliptic curve given by the lattice $\mathbb{Z} + \mathbb{Z}z$, are linear combinations of theta series. Indeed,

$$\Delta(z) = \frac{691}{265344}(2\pi)^{12}(\theta_1(z) - \theta_2(z)), \tag{1}$$

where $\theta_1(z)$ and $\theta_2(z)$ are theta series attached to the quadratic lattices Γ_{24} and $\Gamma_8 \oplus \Gamma_8 \oplus \Gamma_8$ respectively, see [Se]. Since the Fourier coefficients of $\Delta(z)$ are the famous Ramanujan partition numbers $\tau(n)$, (1) gives intrinsic relations between the partition numbers and the numbers of lattice points of certain lengths. This naturally leads to the so-called basis problem: Which modular forms are linear combinations of theta series and how to express them? In the light of Klingen's work on the decomposition of the space of modular forms as direct sums of subspaces of Eisenstein liftings of cuspforms (see [Kl] for the case of Siegel modular forms), the problem is reduced to a problem regarding cuspforms.

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