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THE THERMODYNAMIC FORMALISM APPROACH TO SELBERG'S ZETA FUNCTION FOR PSL(2, Z)

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I. INTRODUCTION

Besides the classical approach to Selberg's zeta function for cofinite Fuchsian groups [S] through the trace formula [V] there has been developed recently another one based on the thermodynamic formalism [R2] applied to the dynamical zeta function of Smale and Ruelle [F] which for geodesic flows on surfaces of constant negative curvature (c.n.c.) is closely related to Selberg's function for the corresponding Fuchsian group [Sm, R1]. This latter approach however has been worked out up to now only for cocompact groups.

In this announcement we discuss the first example of a cofinite, noncocompact Fuchsian group where the aforementioned thermodynamic formalism approach works also, namely the modular group $\mathrm{PSL}(2, \mathbf{Z})$. The most remarkable fact with this group is that the whole formalism can be made rather explicit contrary to the general case where many of the constructions used are rather difficult to come by. The reason for this is a quite simple construction of symbolic dynamics for geodesic flows on surfaces of c.n.c. due to Bowen and Series [BS]. Instead of an usually only inductively defined Markov partition [F] their symbolic dynamics is based on a piecewise analytic Markov map of the limit set of the Fuchsian group, determined by the group generators. Through this symbolic dynamics the Smale-Ruelle function for the flow gets transformed into a generating function for partition functions for the B-S map to which the transfer operator method of statistical mechanics applies [M1, R1]. Since for cocompact groups the B-S maps are expanding [BS] their transfer operators can be chosen as nuclear operators [G], and the Selberg function finally gets

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