BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Algebraic structures of bounded symmetric domains, by I. Satake. Iwanami Shoten and Princeton University Press, 1980, x + 317 pp. ISBN 0-691-08271-5

This is an important book because it is the first one discussing the general theory of bounded symmetric domains under all its important aspects. This may sound surprising given the age and importance of the subject; the explanation lies probably in its difficulty and in its rich and complicated relations with other fields.

To describe the subject briefly, consider a bounded domain D in $\mathbf{C}^n$. Let $\text{Aut}(D)$ denote the group of all holomorphic bijections of D onto itself; this is a Lie group (e.g., because it is a closed subgroup of the isometry group of D in the Bergman metric). D is said to be *homogeneous* if for all z, w in D, there is a g in $\text{Aut}(D)$ with $g(z) = w$. D is *symmetric* if there is always a g with $g(z) = w$ *and* $g(w) = z$. (With the Bergman metric such a D is a Riemannian, even Hermitian, symmetric space; hence $\text{Aut}(D)$ is semisimple.... .) When $n = 1$, the unit disc and every simply connected domain are symmetric. When $n > 1$, there is no Riemann mapping theorem, and homogeneous domains become quite rare. Symmetric domains are even rarer: For every n, there are only finitely many nonequivalent ones (due to a similar situation for semisimple Lie groups). They are completely classified. There are five (or four depending on how one counts) infinite classes called the classical domains; they correspond to certain Lie groups of classical type, and there are two "exceptional domains," one in $\mathbf{C}^{16}$ and one in $\mathbf{C}^{27}$. E. Cartan did the classification in 1935 up to the proof of existence of the exceptional domains; this