BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

ENDS OF RIEMANNIAN MANIFOLDS WITH NONNEGATIVE RICCI CURVATURE OUTSIDE A COMPACT SET

MINGLIANG CAI

ABSTRACT. We consider complete manifolds with Ricci curvature nonnegative outside a compact set and prove that the number of ends of such a manifold is finite and in particular, we give an explicit upper bound for the number.

1. INTRODUCTION

Toponogov [T] showed that in a complete manifold of nonnegative sectional curvature, a line splits off isometrically, i.e. any nonnegatively curved M^n is isometric to a Riemannian product $N^k \times R^{n-k}$, where N^k does not contain a line. Later, Cheeger and Gromoll [CG] generalized this to manifolds of nonnegative Ricci curvature, known as the Cheeger-Gromoll splitting theorem. As a consequence, such a manifold has at most two ends (see §2 for the definition of an end). In [A], Abresch studied manifolds with asymptotically nonnegative sectional curvature. He showed that the number of ends of such a manifold is finite and can be estimated from above explicitely. In this note, we consider manifolds with Ricci curvature being nonnegative outside a compact set and prove that the number of ends of such a manifold is finite and in particular, we give an explicit upper bound for the number. That is, we prove the following theorem.

Theorem. *Let (M^n, o) be a Riemannian manifold with base point o. If the Ricci curvature is nonnegative outside the geodesic ball $B(o, a)$ of radius a and is bounded from below on $B(o, a)$ by $-(n-1)\Lambda^2$ (for $\Lambda \geq 0$), then there exists a universal bound on the number of ends, e.g.*

$$\textit{the number of ends of } M^n \leq \frac{2n}{n-1}(\Lambda a)^{-n} \exp\left(\frac{17(n-1)}{2}\Lambda a\right).$$

Received by the editors September 25, 1990 and, in revised form, October 9, 1990.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 53C20.