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A COMPLETE SOLUTION TO THE POLYNOMIAL 3-PRIMES PROBLEM

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I. INTRODUCTION

By the "classical 3-primes problem" we mean: *can every odd number ≥ 7 be written as a sum of three prime numbers*? This problem was attacked with spectacular success by Hardy and Littlewood [8] in 1923. Using their famous *Circle Method* and assuming the Generalized Riemann Hypothesis, they proved that there exists a positive number N such that every odd integer $n \geq N$ is a sum of three primes. In 1937, Vinogradov [12] employed his ingenious methods for estimating exponential sums to prove the Hardy-Littlewood conclusion without invoking the Riemann Hypothesis. The result is therefore known as Vinogradov's Theorem. Vinogradov's proof actually implies a computable value for N, raising the possibility that the classical 3-primes problem can be completely settled by computation. For example, by carefully estimating the errors in Vinogradov's proof, Borodzkin [2] showed that one can take

$$N = 3^{3^{15}}.$$

Unfortunately, this value is far beyond the minimum that would make the problem accessible to even the fastest computers.

If instead of $\mathbf{Z}$ we consider the ring $\mathbf{F}_q[x]$ of polynomials in a single variable x over the finite field $\mathbf{F}_q$ of q elements, we can easily formulate, in direct analogy to the classical 3-primes problem, a *polynomial* 3-*primes problem*. To this end we observe that the analog of prime number is irreducible polynomial, of positive number is monic polynomial, and we need also:

Definition. A monic polynomial M over $\mathbf{F}_q$ is called *even* if $q = 2$ and if M is divisible by x or $x + 1$; otherwise M is called *odd* (so, for all $q \neq 2$, all M are odd).

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