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ON THE LEBESGUE MEASURABILITY OF CONTINUOUS FUNCTIONS IN CONSTRUCTIVE ANALYSIS

DOUGLAS BRIDGES AND OSVALD DEMUTH

ABSTRACT. The paper opens with a discussion of the distinction between the classical and the constructive notions of "computable function." There then follows a description of the three main varieties of modern constructive mathematics: Bishop's constructive mathematics, the recursive constructive mathematics of the Russian School, and Brouwer's intuitionistic mathematics. The main purpose of the paper is to prove the independence, relative to Bishop's constructive mathematics, of each of the following statements:

There exists a bounded, pointwise continuous map of $[0, 1]$ into $\mathbf{R}$ that is not Lebesgue measurable.

If μ is a positive measure on a locally compact space, then every real-valued map defined on a full set is measurable with respect to μ.

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The purpose of this article is to answer the following question within a framework which makes the discussion accessible to mathematicians who know little or nothing about the foundational technicalities of modern constructive mathematics:

> *In constructive mathematics, are there any real-valued functions defined on* $[0, 1]$ *that are not Lebesgue measurable?*

As we shall see, the answer to this question depends on the variety of constructive mathematics within which measure theory is developed.

We first explain the difference between constructive mathematics and the "classical" mathematics practised by the majority of our colleagues today, and describe the three varieties of constructive mathematics with which this paper is concerned. The reader

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The second author is deceased.