BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

0273-0979/91 $1.00 + $.25 per page

Fibrewise topology, by I. M. James. Cambridge University Press, Cambridge, New York, New Rochelle, Melbourne, Sydney, 1989, x+198 pp., $49.50. ISBN 0-521-36090-0

Fibrewise topology can be thought of as the topology of continuous families of spaces or maps. The objects then of this category are space-valued functions $E(b)$ of a parameter b which varies in a topological space B. For instance, if we assign to every linear map $b : R^m \to R^n$ its image, then $E(b) = \mathrm{im}(b)$ is such a function, $B = \mathscr{L}(R^m, R^n)$. This example has obvious continuity properties, but in general it is not so clear what continuity of $E(b)$ should mean. One standard procedure is to form the set $E = \{(b, x)|b \in B, x \in E(b)\}$ and topologize this set in such a way that the projection $p : E \to B$, $p(b, x) = b$ is continuous. This makes good sense if all $E(b)$ are contained in one big space $\mathscr{X}$, so that $E \subset B \times \mathscr{X}$. Even if B is not topologized to begin with, the fact or choice of having all $E(b)$ contained in one space $\mathscr{X}$ often suggests or induces an appropriate topology on B. (Example: If M is a compact smooth manifold, let B be the set of all submanifolds of R^n, $n \leq \infty$, which are diffeomorphic to M, and put $E(b) = b$.)

The book under review takes the result of this procedure as the starting point, i.e. a continuous map $p : E \to B$. This is now called a *fibrewise topological space over* B—and $E(b) = p^{-1}(b)$ can be thought of as a continuous family of spaces, $b \in B$. The parameter space B is called the *base* space, $E(b) = p^{-1}(b)$ is the *fibre* over b, and E itself may be called a fibrewise topological space over B—if the projection is understood from the context. A morphism between two objects is a continuous family $\varphi(b) : E(B) \to E'(b)$ of continuous maps, i.e. a continuous map $\varphi : E \to E'$, such that $p'\varphi = p$; it is called a *fibrewise continuous map* over B. The book is very consistent in its fibrewise thinking and fibrewise language. As the author remarks, "the effect is somewhat monotonous $\cdots$ but experience shows that to compromise on this point is liable to cause confusion." One can agree with this cautious, careful attitude, but the reviewer feels that the functional (parametric) point of view could have been used simultaneously, perhaps informally, to facilitate the understanding or to stimulate