

8. R. Smith, *The Riemann problem in gas dynamics*, Trans. Amer. Math. Soc. **249** (1979), 1–50.

J. A. SMOLLER
UNIVERSITY OF MICHIGAN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991
©1991 American Mathematical Society
0273-0979/91 \$1.00 + \$.25 per page

Modular forms, by T. Miyake. Springer-Verlag, Berlin, New York, 1989, 335 pp., \$73.00. ISBN 3-540-50268-8

Modular forms have been studied, *accidentally or intentionally*, for about 200 years, beginning seriously with Jacobi and Eisenstein. A key word here is “accidentally”: Historically, many peculiar things were discovered and studied in an *ad hoc* fashion; a great number of these are now construed as corollaries of a general phenomenology with the unfortunately unevocative appellations “theory of modular forms” or “theory of automorphic forms.” This “underlying phenomenology” is distant from more tangible and elementary issues, and so often seems obscurely technical and tiresomely unmotivated (to the uninitiated, at least).

Because it does provide an underlying pattern, the subject is currently of intense research interest. Either provably or conjecturally, a large fraction of the objects of interest in number theory is intimately related to modular forms. There are also pleasantly surprising connections with many other things: string theory, combinatorics, Kac-Moody algebras, and so on.

To develop a sympathy for the subject, it seems necessary to shift what one believes to be the *primary objects of study*. Because of the efficacy of “the theory of modular forms” as a methodology in number theory, one might study modular forms as fundamental objects, rather than directly consider number fields themselves (for example). To add to the confusion of the novice, there is not a single fixed notion of “modular form”: The general idea has many different incarnations, whose common spirit is apparent only after considerable reflection (and proof). Yet, each incarnation of “modular form” has its own utility.

With just a few exceptions, the subject languished for the first half of this century. Its resurrection in the late 1950s and early