BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

AVERAGE CASE COMPLEXITY OF MULTIVARIATE INTEGRATION

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ABSTRACT. We study the average case complexity of multivariate integration for the class of continuous functions of d variables equipped with the classical Wiener sheet measure. To derive the average case complexity one needs to obtain optimal sample points. This optimal design problem has long been open. All known designs guaranteeing average case error ε lead to an exponential number of sample points, roughly $\Theta(\varepsilon^{-d})$. For d large this makes the problem intractable for even the fastest computers.

Yet good designs have to exist since the average case complexity is bounded by $O(\varepsilon^{-2})$ as can be proven by considering the Monte-Carlo algorithm. We just did not know how to construct them.

In this paper we prove that optimal design is closely related to discrepancy theory which has been extensively studied for many years. Of particular importance for our purpose are papers by Roth [10, 11]. This relation enables us to show that optimal sample points can be derived from Hammersley points. Extending the result of Roth [10] and using the recent result of Wasilkowski [19], we conclude that the average case complexity is $\Theta(\varepsilon^{-1}(\ln \varepsilon^{-1})^{(d-1)/2})$.

1. INTRODUCTION

The approximate computation of multivariate integrals has been extensively studied in many papers, see [5–7, 17] for hundreds of references. We assume that multivariate integrals are approximated by evaluating integrands at finitely many sample points and by performing arithmetic operations and comparisons on real numbers. Assume that the cost of one integrand evaluation is $\mathbf{c}$, and that the cost of one arithmetic operation or comparison is taken as 1. Usually $\mathbf{c} \gg 1$.

Received by the editors September 28, 1989.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 65D30, 41A55.

This research was supported in part by the National Science Foundation under Contract IRI-89-07215.