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MINIMAL SURFACES BOUNDED BY A PAIR OF CONVEX PLANAR CURVES

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In 1956 M. Shiffman [9] proved several beautiful theorems concerning the geometry of a minimal annulus A whose boundary consists of two closed smooth convex curves in parallel planes P_1, P_2. The first theorem stated that the intersection of A with any plane P, between P_1 and P_2, is a convex Jordan curve. In particular it follows that A is embedded. He then used this convexity theorem to prove that every symmetry of the boundary of A extends to a symmetry of A. In the case that ∂A consists of two circles Shiffman proved that A is foliated by circles in parallel planes. Earlier Riemann [7] described, in terms of elliptic functions, all minimal annuli that can be foliated by circles in parallel planes (also see [2] for an analytic description of these surfaces as well as a computer graphics image of one of them). Together these results of Riemann and Shiffman yield a classification of all minimal annuli with boundary consisting of circles in parallel planes.

We shall call a compact minimal surface M *stable* if, with respect to any nontrivial normal variation fixing the boundary, the second derivative of area is positive. If the second derivative of area is negative for some variation, then M is called *unstable.* If M is neither stable nor unstable, we will call it *almost-stable.*

The theorem given below augments Shiffman's first theorem.

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