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BOUNDED CURVATURE CLOSURE OF THE SET OF COMPACT RIEMANNIAN MANIFOLDS

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1. Introduction

In this note we consider the set of metric spaces which are the limits with respect to Lipschitz distance d_L of compact connected C^∞-Riemannian manifolds of curvature uniformly bounded above and below. We call this set "bounded curvature closure" (BCC).

It is well known that the limit spaces need not be C^2-Riemannian manifolds [P, Example 1.8]. Hence, the problem arising is to give a geometrical description of the BCC.

We solve this problem with the help of the theory of metric spaces of bounded curvature which A. D. Aleksandrov introduced more than 30 years ago [A] to construct the synthetic generalization of Riemannian geometry. Our principal result (Closure Theorem) states that the BCC consists of all compact Aleksandrov's spaces of bounded curvature.

A consideration of the BCC is definitely of independent interest, but due to Gromov's compactness theorem it has proved to be useful to consider the BCC in connection with different problems of Riemannian geometry. Now we are going to formulate the compactness theorem and explain the ways of its applications. First let us give necessary definitions.

Let $(\mathscr{M}_1, \rho_1)$, $(\mathscr{M}_2, \rho_2)$ be metric spaces with metrics ρ_1 and ρ_2, $f: (\mathscr{M}_1, \rho_1) \to (\mathscr{M}_2, \rho_2)$ a Lipschitz map. Then

$$\operatorname{dil} f = \sup\{\rho_2(f(x), f(y))/\rho_1(x, y) | x, y \in \mathscr{M}_1, x \neq y\}$$

is called *dilatation* of f.

Suppose that $(\mathscr{M}_1, \rho_1)$, $(\mathscr{M}_2, \rho_2)$ are compact. Then

$$d_L(\mathscr{M}_1, \mathscr{M}_2) = \inf\{|\ln \operatorname{dil} f| + |\ln \operatorname{dil} f^{-1}| \,|f \text{ bi-Lipsch.hom.}\}$$

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