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ABSOLUTE INTEGRAL CLOSURES ARE BIG COHEN–MACAULAY ALGEBRAS IN CHARACTERISTIC P

MELVIN HOCHSTER AND CRAIG HUNEKE

Throughout this paper "ring" means commutative ring with identity and modules are unital. Our main interest is in local rings, i.e., Noetherian rings (R, m) with a unique maximal ideal m. In such a ring, $x_1, \dots, x_n \in m$ is a *system of parameters* if m has a power in the ideal $(x_1, \dots, x_n)R$ and n is the Krull dimension of R. When R is complete and contains a field, this means that R is module-finite over a formal power series subring $K[[x_1, \dots, x_n]] = A$. R is called *Cohen–Macaulay* if some (equivalently, every) system of parameters is a regular sequence in R, which means that every x_{i+1} is a nonzerodivisor on $R/(x_1, \dots, x_i)R$, for $0 \le i \le n-1$. In the case where R is module-finite over the formal power series subring A, this means that R is a free A-module. For many theorems of commutative algebra and of algebraic geometry, the Cohen–Macaulay condition (possibly on the local rings of a variety) is just what is needed to make the theory work. Our main result, which is given below in an algebraic form in Theorem 1 and in a geometric form in Theorem 2, asserts that, under mild conditions on a local ring R of positive prime characteristic p, one can "correct" the failure of the Cohen–Macaulay condition in R itself by passing to a very large integral extension of R. It is worth emphasizing that this is quite false in characteristic 0, and comes as a surprise, we believe, in characteristic p.

One can get an idea of how far these theorems are from the truth over a field of characteristic 0 from the following observation: In the situation of Theorem 2, when X is projectively normal, the maps of cohomology are always injective, by an easy trace argument. Despite the fact that both Theorem 1 and Theorem 2

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