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NORMAL SUBGROUPS OF SL(2, A)

DOUGLAS COSTA AND GORDON KELLER

0. Introduction

We announce a characterization of the normal subgroups of $\mathrm{SL}(2, A)$ for a large class of commutative rings A including all arithmetic Dedekind domains with infinitely many units and all rings satisfying the "smallest" stable range condition (see (3.2)).

Our characterization requires several new definitions and results valid for $\mathrm{SL}(2, A)$ with A an arbitrary commutative ring. These results provide new tools for studying normal subgroups of $\mathrm{SL}(2, A)$ and should prove to be of general interest. Our main theorem extends the work of Bass, Milnor, and Serre [2] for $n \geq 3$ to the case $n = 2$ for as large a class of rings as one could reasonably expect.

If A is a commutative ring, $\mathrm{SL}(n, A)$ denotes the $n \times n$ matrices with entries in A and determinant 1. If J is an ideal in A, $L(n, A; J)$ is the normal subgroup of $\mathrm{SL}(n, A)$ consisting of those matrices congruent to scalars mod J, and $\mathrm{SL}(n, A; J)$ consists of those elements of $\mathrm{SL}(n, A)$ congruent to the identity mod J. The elementary matrix $E_{ij}(x)$, $i \neq j$, is that $n \times n$ matrix agreeing with the identity matrix except for the i, j position whose entry is x. Let $E(n, A)$ denote the subgroup of $\mathrm{SL}(n, A)$ generated by elementary matrices. If S is a subset of A, $E(n, A; S)$ is the smallest normal subgroup of $E(n, A)$ containing all elementary matrices whose nonzero off-diagonal entry comes from S.

Bass, Milnor, and Serre [2] concluded that for $n \geq 3$ and A an arithmetic Dedekind domain, N is normal in $\mathrm{SL}(n, A)$ if and only if $E(n, A; J) \subseteq N \subseteq L(n, A; J)$ for some ideal J. Furthermore, $[L(n, A; J), \mathrm{SL}(n, A)] = E(n, A; J)$. (If H and

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