BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

ON THE DISCRETE SPECTRUM OF CERTAIN DISCRETE GROUPS

ISAAC EFRAT

Our purpose in this note is to announce certain results in the harmonic analysis of discrete cofinite subgroups that act on the building attached to the group $PGL_2(K)$, where K is a local field over a function field. These results all stand in sharp contrast to known theorems for the subfamily of congruence subgroups, and settle the function field case of conjectures made in the analogous context of $SL_2(\mathbb{R})$.

This basic setup in which our problem arises (see [Sel]) is that of a discrete subgroup Γ acting on the upper half-plane $\mathbb{H} = SL_2(\mathbb{R})/SO_2(\mathbb{R})$, so that the quotient is of finite volume but not compact, that is, a surface with finitely many cusps. One then forms the space $L^2(\Gamma\backslash\mathbb{H})$ of Γ-automorphic $\mathbb{C}$-valued functions that are square integrable on this quotient, and studies its decomposition into subspaces spanned by discrete and continuous spectra of the invariant differential operators, which here are generated by the Laplacian. Of particular interest are the *cusp forms,* which are the discrete eigenfunctions that decay rapidly at all cusps, and which include those whose eigenvalues are imbedded inside the continuous spectrum (the latter is constructed using *Eisenstein series*). The question is then whether such cusp forms actually exist, and if they do, whether their contribution to the space of square integrable functions is in a suitable sense maximal.

When Γ is a *congruence subgroup,* all discrete eigenfunctions (except for the constants) are cuspidal, and these occur abundantly. Indeed if we parametrize the discrete spectrum by the Laplace eigenvalues $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \dots$ and let $N_\Gamma(T)$ stand for the number of these in $[0, T]$, then it is known that

$$(1) \qquad N_\Gamma(T) \sim cT \quad \text{as } T \to \infty,$$

Received by the editors June 22, 1989 and, in revised form, May 22, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 11F12, 11F72, 11F85.
Partially supported by the Alfred P. Sloan and the National Science Foundations.

0273-0979/91 $1.00 + $.25 per page