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THE WIENER LEMMA AND CERTAIN OF ITS GENERALIZATIONS

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Let Γ be the unit circle in the complex plane. Let $\mathscr{A}$ be the Banach algebra of all complex valued continuous functions on Γ with absolutely convergent Fourier series.

Norbert Wiener [10] proved that if any f in $\mathscr{A}$ is invertible in the ring of continuous functions on Γ, then $1/f$ also is an element of $\mathscr{A}$. Paul Lévy [7] generalized Wiener's result, showing that for each f in $\mathscr{A}$ and each complex analytic function Φ that is defined on a neighborhood of $f(\Gamma)$, $\Phi(f)$ belongs to $\mathscr{A}$.

Lévy did so by an argument that shows, more generally, that $\Phi(f)$ belongs to $\mathscr{A}$ whenever f is in $\mathscr{A}^d$ and Φ is analytic on some neighborhood of $f(\Gamma)$ in $\mathbf{C}^d$. Later, G. E. Šilov [9] established such a result for a class of Banach algebras of continuous functions that includes $\mathscr{A}$. Šilov's proof uses the Cauchy-Weil integral formula for an analytic function of several complex variables.

1. An integral formula

Here, we prove the several variable form of the Wiener-Lévy theorem by showing that for each f in $\mathscr{A}^d$ and each Φ analytic on a neighborhood of $f(\Gamma)$ in $\mathbf{C}^d$, $\Phi(f)$ is given by a one variable Cauchy integral formula for a related $\mathscr{A}$-valued function that is analytic on a neighborhood of Γ in the plane.

Let u be the identity function restricted to Γ, viewed as an element of $\mathscr{A}$. If z is at a positive distance from Γ, either $1/z$ times the geometric series in u/z or $-1/u$ times the geometric series in z/u is an inverse for $z - u$ in $\mathscr{A}$. Therefore, as an $\mathscr{A}$-valued function of z, $1/(z-u)$ is uniformly continuous on the complement of each neighborhood of Γ.

For each n, let f_n be the mapping from Γ to $\mathbf{C}^d$ each coordinate of which is the truncation from $-n$ to n of the Fourier series

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