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A SIMPLIFIED VERSION OF THE ABSTRACT CAUCHY-KOWALEWSKI THEOREM WITH WEAK SINGULARITIES

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ABSTRACT. A simplified version of the abstract Cauchy-Kowalewski theorem of Nirenberg [9], Ovsjannikov [12], Nishida [10], Baouendi and Goulaouic [3], and Asano [1] is presented. The new version requires more specific information on the form of the equation but recovers a stronger result in that the region of existence is not forced to shrink at each step of an iteration and that weak singularities are allowed along the boundary of the region of existence.

The Cauchy–Kowalewski theorem is the basic existence theorem for analytic solutions of partial differential equations and in its abstract form [1, 3, 9, 10, 12] can be applied to equations that involve nonlocal operators, such as the water wave equations [8], the Boltzmann equation in the fluid dynamic limit [11], the incompressible fluid equations in the zero-viscosity limit [2] and the vortex sheet equations [4–6, 13]. The proof of the abstract Cauchy–Kowalewski theorem in [1, 3, 9, 10, 12] is of "Nash–Moser type" in that it requires a loss in the size of the existence region at each step of an iteration (but without use of Newton iteration).

The purpose of this paper is to present a new version of the abstract Cauchy–Kowalewski theorem that does not use a "Nash–Moser type" proof. The domain of existence does not shrink at each step of the iteration, and in addition mild singularities are allowed at the boundary of the existence region. The main significance of this new version is that the simpler proof makes it more adaptable to new applications and that more control over the region of existence allows solution of problems with singularities.

The new version of the theorem does require more information on the structure of the equation. However these new hypothe-

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