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ON THE COMPLETE INTEGRABILITY OF SOME LAX SYSTEMS ON $\mathrm{GL}(n, R) \times \mathrm{GL}(n, R)$

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INTRODUCTION

Over the past decade there has been a great deal of activity in the solution of nonlinear evolution equations by the Riemann problem method (see [5] and references therein). As is well known, at the basis of all these works is the representation of the equations as a condition of zero curvature, i.e.,

$$\frac{\partial U}{\partial t} - \frac{\partial V}{\partial x} + [U, V] = 0. \tag{1}$$

Here, U and V are matrix-valued functions parametrized by the classical fields and $[\cdot, \cdot]$ is the standard commutator. For periodic lattice models, where the (discretized) spatial variable n now takes values in $\mathbf{Z}_N = \mathbf{Z}/N\mathbf{Z}$, there is a natural analog of (1). These are the so-called Lax systems [1, 5] and have the general form

$$\frac{dL_n}{dt} = V_{n+1}L_n - L_n V_n, \qquad n \in \mathbf{Z}_N. \tag{2}$$

The matrices L_n above are invertible and define parallel transport from site n of the lattice to site $n+1$ [5]. As can be easily verified, the monodromy matrix $T(L) = L_N \cdots L_1$, $L = (L_1, \ldots, L_N)$ undergoes an isospectral deformation

$$\frac{dT(L)}{dt} = [V_1, T(L)] \tag{3}$$

and hence the eigenvalues of $T(L)$ provide a collection of conserved quantities for (2). In this note, we shall consider a special case of (2) which is related to eigenvalue algorithms and for which additional integrals can be constructed to prove complete integrability (in the sense of Liouville) on generic symplectic leaves. For the convenience of the general reader, we recall that a Poisson

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