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A NEW FORMULATION OF THE GENERALIZED TODA LATTICE EQUATIONS AND THEIR FIXED POINT ANALYSIS VIA THE MOMENTUM MAP

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INTRODUCTION

The Toda flows are examples of integrable Hamiltonian systems which are of great interest from the point of view of pure mathematics as well as in applications. They originated as a description of a 1-dimensional lattice of particles with exponential interaction [29]. Flaschka [14] found a Lax pair for the equations and Moser [25] analyzed the dynamics and scattering behavior of the system. Adler [1], Kostant [21,22] and Symes [26,27] generalized the results to arbitrary Lie algebras. Symes [28] also showed that the Toda equations have a surprising connection with numerical analysis: the time-1 map of the Toda flow is equivalent to the QR algorithm for diagonalizing symmetric matrices (see also [11,13,17,23,28]).

Our point of view in this paper also originated from analyzing a problem of numerical importance. In [8, 9] Brockett studied a critical point problem arising from geometrical matching problems in computer vision. He derived the gradient flow associated with the critical point problem and showed that the equations had an elegant Lax pair form. In the case of the orthogonal group these equations give a new method of solving the symmetric eigenvalue problem. In [4] Bloch observed that a particular case of these equations gives the Toda equations in the Flaschka form.

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