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ANTIMONOTONICITY: CONCURRENT CREATION AND ANNIHILATION OF PERIODIC ORBITS

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ABSTRACT. One-parameter families f_λ of diffeomorphisms of the Euclidean plane are known to have a complicated bifurcation pattern as λ varies near certain values, namely where homoclinic tangencies are created. We argue that the bifurcation pattern is much more irregular than previously reported. Our results contrast with the monotonicity result for the well-understood one-dimensional family $g_\lambda(x) = \lambda x(1-x)$, where it is known that periodic orbits are created and never annihilated as λ increases. We show that this monotonicity in the creation of periodic orbits never occurs for any one-parameter family of C^3 area contracting diffeomorphisms of the Euclidean plane, excluding certain technical degenerate cases where our analysis breaks down. It has been shown that in each neighborhood of a parameter value at which a homoclinic tangency occurs, there are either infinitely many parameter values at which periodic orbits are created or infinitely many at which periodic orbits are annihilated. We show that there are *both* infinitely many values at which periodic orbits are *created* and infinitely many at which periodic orbits are *annihilated*. We call this phenomenon *antimonotonicity*.

I. INTRODUCTION

The orbit of point x under a diffeomorphism of the plane f is the sequence $\{f^k(x)\}$, where for $k \geq 0$, f^k denotes the k-fold composition of f, f^{-k} denotes the k-fold composition of f^{-1} and f^0 is the identity map. Let p be a periodic point with period n. The stable manifold $W^s(p)$ of the point p is the set $\{x : \lim_{k\to\infty} f^{nk}(x) = p\}$. Similarly, the unstable manifold $W^u(p)$ of p is $\{x : \lim_{k\to\infty} f^{-nk} = p\}$. We assume that p is a hyperbolic saddle, that is, the eigenvalues e_1, e_2 of $Df^n(p)$ are such that $|e_1| < 1 < |e_2|$. Since f is a diffeomorphism of the plane, both $W^s(p)$ and $W^u(p)$ are curves. There exists a homoclinic tangency

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