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TRIANGULAR ALGEBRAS AND IDEALS OF NEST ALGEBRAS

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1. Introduction

Let $\mathscr{H}$ be a separable Hilbert space and $\mathscr{T} \subseteq \mathscr{B}(\mathscr{H})$ be an algebra of bounded operators. Say $\mathscr{T}$ is *triangular* if $\mathscr{T} \cap \mathscr{T}^*$ is a maximal abelian self-adjoint subalgebra (m.a.s.a.) of $\mathscr{B}(\mathscr{H})$ and call this m.a.s.a. the *diagonal* of $\mathscr{T}$. A triangular algebra is *maximal* triangular if it is not properly contained in any triangular algebra. Triangular algebras of operators have been studied for 30 years now, since the seminal paper of Kadison and Singer [6]. In this, they proposed the maximal triangular algebras as infinite-dimensional generalizations of the upper triangular matrices and as the non-self-adjoint analogues of the von Neumann algebras. However, general questions on maximal triangular algebras have proved highly intractible and a theory of non-self-adjoint algebras based on these algebras has not developed. Nevertheless, special classes of triangular algebras have provided much of the motivation in the subsequent study of non-self-adjoint algebras, in the areas of nest algebras and CSL algebras (see [3]), algebras connected with ergodic actions [1] and non-self-adjoint subalgebras of certain C^*-algebras and von Neumann algebras [10, 11]. Specifically, the study of nest algebras is now well developed and, as we show here, it is now possible to use the techniques of this subject to answer some of the hard questions which arose early in the study of triangular algebras.

In particular, it was hoped that, just as Murray and von Neumann essentially reduced the study of von Neumann algebras to the study of factors, analogous "primitive" maximal triangular

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