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FINITELY GENERATED GROUPS, p-ADIC ANALYTIC GROUPS, AND POINCARÉ SERIES

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INTRODUCTION

Igusa [I 1, I 2] was the first to exploit p-adic integration with respect to the Haar measure on $\mathbf{Q}_p$ in the study of Poincaré series arising in number theory and developed a method using Hironaka's resolution of singularities to evaluate a limited class of such integrals. Denef [D 1, D 2] and, more recently, Denef and van den Dries [DvdD] have applied results from logic, profiting from the flexibility of the concept of definable, greatly to enlarge the class of integrals amenable to Igusa's method. In [DvdD] these results are employed to answer questions posed by Serre [S] and Oesterlé [O] concerning the rationality of various Poincaré series associated with the p-adic points of a closed analytic subset of $\mathbf{Z}_p^m$. In this note we apply these techniques to prove that various Poincaré series associated with finitely generated groups and p-adic analytic groups are rational in p^{-s}, extending results of [GSS].

RESULTS

Let G be a group and denote by $a_n(G)$ the number of subgroups of index n in G. We are interested in groups for which $a_n(G)$ is finite for every $n \in \mathbf{N}$. For each prime p, we can then associate the following Poincaré series with this arithmetical function:

$$(1) \qquad \zeta_{G,p}(s) = \sum_{n=0}^{\infty} a_{p^n}(G) p^{-ns} = \sum_{H \in \mathbf{X}_p} |G : H|^{-s}$$

where $\mathbf{X}_p = \{H \leq G : H \text{ has finite } p\text{-power index in } G\}$.

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