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A NONLINEAR EXTENSION OF THE BOREL DENSITY THEOREM: APPLICATIONS TO INVARIANCE OF GEOMETRIC STRUCTURES AND TO SMOOTH ORBIT EQUIVALENCE

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Let G be a connected semisimple Lie group with no compact factors and finite center and let Γ be a lattice in G (i.e. a discrete subgroup such that G/Γ has a finite invariant measure). Let π be a representation of G on some vector space V. Borel [Bo] proved that if π is a rational representation and V is finite dimensional then every Γ-invariant line in V is G-invariant; in fact, this is equivalent to saying that Γ is Zariski dense in G. On the other hand, if we allow V to be infinite dimensional but we require π to be unitary, Moore [M] proved that every Γ-invariant vector $v \in V$ is G-invariant; this is true for Γ not necessarily discrete, as long as it is not compact. However, the same result is far from being true for any infinite dimensional representation. Here we announce the proof of an extension of Borel's theorem in two different directions: one involving nonlinear actions, the other involving some particular infinite dimensional linear representations which arise naturally from purely geometric considerations.

Let G, Γ be as above with the further assumption that Γ is irreducible (i.e. we want to eliminate the case in which $\Gamma = \Gamma_1 \times \Gamma_2 \subset G_1 \times G_2 = G$) and let H be a real algebraic group. Let M be a smooth manifold and let $P \to M$ be a principal H-bundle on which G acts by automorphisms. Suppose that X consists of the real points of a variety defined over $\mathbf{R}$ on which H acts algebraically and let $E \to M$ be the bundle with fiber X associated to P. Then we have the following result:

Theorem 1. *If every orbit in M with compact stabilizer is not locally closed, then every measurable Γ-invariant section of E is G-invariant.*

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