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DEFORMATION RIGIDITY FOR SUBGROUPS OF $SL(n, \mathbf{Z})$ ACTING ON THE n-TORUS

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ABSTRACT. We announce and give a sketch of the proof of the result:

Theorem 1. *For $n \geq 3$, the standard action of $SL(n, \mathbf{Z})$ on $\mathbf{T}^n$ is smoothly and analytically rigid under C^0-deformations.*

Several related results concerning rigidity of actions of subgroups of $SL(n, \mathbf{Z})$ on $\mathbf{T}^n$ that follow from our method are also discussed.

§1. RIGIDITY THEOREM

The natural action of the determinant-one, integer $n \times n$-matrices $SL(n, \mathbf{Z})$ on $\mathbf{R}^n$ preserves the integer lattice $\mathbf{Z}^n$; hence for each subgroup $\Gamma \subset SL(n, \mathbf{Z})$ there is an induced standard action on the quotient n-torus, $\varphi: \Gamma \times \mathbf{T}^n \to \mathbf{T}^n$. A basic problem is to understand the smooth actions near to such a standard action in terms of their geometry and dynamics (cf. [8, 19]). In this note we announce results which classify 1-parameter deformations of standard actions.

A C^k-deformation of φ is a 1-parameter family of C^∞-actions $\varphi_t: \Gamma \times \mathbf{T}^n \to \mathbf{T}^n$, $0 \leq t \leq 1$ such that $\varphi_0 = \varphi$ and for each $\gamma \in \Gamma$, the C^∞-maps $\varphi_t(\gamma)$ depend C^k on the parameter t. That is, $\varphi_t(\gamma)$ is a C^k-path in the Frechet space $\mathrm{Diff}^\infty(\mathbf{T}^n)$. A C^k-deformation is *trivial* if it is implemented by a C^k-family of inner automorphisms of $\mathrm{Diff}^\infty(\mathbf{T}^n)$. That is, there exists a 1-parameter family of C^∞-diffeomorphisms $H_t: \mathbf{T}^n \to \mathbf{T}^n$ which depends C^k on the parameter, and for all $\gamma \in \Gamma$ satisfies

$$H_t^{-1} \circ \varphi_t(\gamma) \circ H_t = \varphi(\gamma); \qquad 0 \leq t \leq 1. \tag{1}$$

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