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NOT ALL LINKS ARE CONCORDANT TO BOUNDARY LINKS

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0. Introduction

A *link* is a smooth, oriented submanifold $L = \{K_1, \dots, K_m\}$ of S^{n+2} which is the ordered disjoint union of m manifolds each piecewise-linearly homeomorphic to the n-sphere (if $m = 1$, L is called a *knot*). Knots and links play an essential role in the classification of manifolds and, in this regard, perhaps the most important equivalence relation on links is that of link concordance. L_0 and L_1 are *concordant* if there is a smooth, oriented submanifold $C = \{C_1, \dots, C_m\}$ of $S^{n+2} \times [0,1]$ which meets the boundary transversely in ∂C, is piecewise-linearly homeomorphic to $L_0 \times [0, 1]$ and meets $S^{n+2} \times \{i\}$ in L_i for $i = 0, 1$. The particular situation which led to the introduction of this equivalence relation and which indicates its importance is as follows. If S is an immersed 2-disk or 2-sphere in a 4-manifold X, x_0 is a singular value and B is a small 4-ball neighborhood of x_0, then $S \cap B$ is a link in S^3. If L were concordant to a link whose components bound disjoint 2-disks in S^3 (the latter is called a *trivial link*) then the singularity at x_0 could be removed. Thus the fundamental problem is to classify (for fixed m, n) the set of concordance classes.

In the mid-1960s M. Kervaire and J. Levine gave an algebraic classification of the high-dimensional ($n > 1$) knot concordance groups [L2]. For even n these are the trivial group and for odd n they are infinitely generated. In a sequence of papers S. Cappell

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