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SPLITTINGS OF SURFACES

RICHARD K. SKORA

Let F be a compact 2-manifold without boundary and with Euler characteristic $\chi(F) < 0$. Only for convenience endow F with a fixed hyperbolic structure, i.e., a discrete, faithful representation of the fundamental group $\pi_1 F$ into the space of isometries of hyperbolic 2-space. *Teichmüller space*, $\mathcal{T}(F)$, is the space of all hyperbolic structures on F divided out by conjugation. W. P. Thurston [Th1] showed that $\mathcal{T}(F)$ admits a compactification as a ball of dimension $-3\chi(F)$. There is a natural identification of the interior of the ball with $\mathcal{T}(F)$ and the boundary of the ball with the space of projective measured geodesic laminations on F (defined below).

J. W. Morgan and P. B. Shalen [MS1, Mo] considered a more general problem. Let Γ be a finitely generated, nonvirtually Abelian group and let $\mathcal{D}_n = \mathcal{D}(\Gamma, \mathrm{Isom}(H^n))$ be the space of discrete, faithful representations of Γ into the group of isometries of hyperbolic n-space divided out by conjugation. They showed that $\mathcal{D}_n$ admits a compactification $\widehat{\mathcal{D}}_n$ where each point of $\widehat{\mathcal{D}}_n - \mathcal{D}_n$ corresponds to a small action of Γ on an $\mathbf{R}$-tree. When $\Gamma = \pi_1 F$ and $n = 2$, they too show that their boundary $\widehat{\mathcal{D}}_n - \mathcal{D}_n$ is homeomorphic to the space of projective measured geodesic laminations on F.

An $\mathbf{R}$-*tree* is a metric space (T, d), such that any two distinct points are joined by a unique arc and every arc is isometric to an interval in $\mathbf{R}$. It is understood that if a group acts on an $\mathbf{R}$-tree, then it acts by isometries and there is no invariant, proper subtree. An action is *small* if the stabilizer of each arc does not contain a free group of rank two.

The above results motivate studying small actions of Γ on $\mathbf{R}$-

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