BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

0273-0979/90 $1.00 + $.25 per page

Local methods in nonlinear differential equations, by Alexander D. Bruno (Translated by William Hovingh and Courtney S. Coleman), Springer Series in Soviet Mathematics, Springer-Verlag, Berlin, Heidelberg, New York, 1989, x + 348 pp., $119.00. ISBN 0-387-18926-2

In a lecture some years ago, Henry McKean remarked that the theory of differential equations really amounts to finding an invertible transformation $y = h(x)$ under which the (nonlinear) system

$$\frac{dx}{dt} = f(x) \tag{1}$$

becomes

$$\frac{dx}{dt} = 1, \tag{2a}$$

or (almost as good):

$$\frac{dx}{dt} = x. \tag{2b}$$

Of course, the problems are that the transformation h may not exist and, even when it does, it will be itself nonlinear and can rarely be found explicitly. Hence the great excitement about "soliton" equations: classes of nonlinear partial differential equations for which h can be constructed (more or less) explicitly and which are therefore completely integrable. Generally, one can at best hope to approximate h locally: in the neighborhood of a particular solution of (1), usually a fixed point. Such methods are the topic of this book, a recent translation and expansion of the 1979 Russian original.

To introduce the basic ideas, suppose that (1) can be written

$$\dot{x} = Ax + F(x), \tag{3}$$

where $F = \mathscr{O}(|x|^2)$ is nonlinear and we have assumed that the vector field vanishes at $x = 0$ (a fixed point). We can also assume that A has been diagonalized or put into Jordan form by a similarity transformation. One seeks a near identity transformation

$$x = y + H(y) \tag{4}$$

defined on a neighborhood of 0. Under (4), (3) becomes

$$\dot{y} = [I + DH(y)]^{-1}[A(h + H(y)) + F(y + H(y))].$$