BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 1, January 1990

THE STABILITY OF MATTER: FROM ATOMS TO STARS

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Why is ordinary matter (e.g., atoms, molecules, people, planets, stars) as stable as it is? Why is it the case, if an atom is thought to be a miniature solar system, that bringing very large numbers of atoms together (say 10^{30}) does not produce a violent explosion? Sometimes explosions do occur, as when stars collapse to form supernovae, but normally matter is well behaved. In short, what is the peculiar mechanics of the elementary particles (electrons and nuclei) that constitute ordinary matter so that the material world can have both rich variety and stability?

The law of motion that governs these particles is the quantum (or wave) mechanics discovered by Schrödinger [SE] in 1926 (with precursors by Bohr, Heisenberg, Sommerfeld and others). Everything we can sense in the material world is governed by this theory and some of its consequences are quite dramatic, e.g., lasers, transistors, computer chips, DNA. (DNA may not appear to be very quantum mechanical, but notice that it consists of a very long, thin, complex structure whose overall length scale is huge compared to the only available characteristic length, namely the size of an atom, and yet it is stable.) But we also see the effects of quantum mechanics, without realizing it, in such mundane facts about stability as that a stone is solid and has a volume which is proportional to its mass, and that bringing two stones together produces nothing more exciting than a bigger stone.

The mathematical proof that quantum mechanics gives rise to the observed stability is not easy because of the strong electric forces among the elementary constituents (electrons and nuclei) of matter. The big breakthrough came in the mid sixties when Dyson and Lenard [DL] showed, by a complicated proof, that stability

Received by the editors August 15, 1989.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 81H99, 81M05, 85A15; Secondary 81C99, 82A15.

This paper was presented as the Sixty-Second Josiah Willard Gibbs Lecture on January 11, 1989 at the 95th annual meeting of the American Mathematical Society in Phoenix, Arizona.

Work partially supported by U.S. National Science Foundation grant PHY-85-15288-A03.