BOOK REVIEWS

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Foundations of infinitesimal stochastic analysis, by K. D. Stroyan and J. M. Bayod. Studies in Logic and the Foundations of Mathematics, vol. 119, North-Holland, Amsterdam, 1986, xii + 478 pp., $47.50. ISBN 0-444-87927-7

Stochastic analysis is currently one of the most active areas of application of nonstandard analysis. This is attested by the copious literature surveyed by Cutland [**C**] and Anderson [**A2**]. Most current work treats probability theory via the Loeb construction [**L**], which obtains standard probability spaces and stochastic processes from nonstandard ones by a rounding off operation. The probability spaces obtained are called Loeb spaces. The book under review is a general account of the measure theory of Loeb spaces and an introduction to the treatment of stochastic processes via Loeb spaces.

Nonstandard analysis is a modern approach to using infinitesimals in analysis, or in mathematics in general, to express limits and notions deriving from limits. For example, in the infinitesimal calculus on the reals,

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$$

can be expressed in the intuitive way, as

$$\text{for all } x \approx 2, \quad \frac{x^2-4}{x-2} \approx 4.$$

The symbol $\approx$ means "infinitely close," i.e. differing by an infinitesimal. Nonstandard analysis was originated by Abraham Robinson and modeled on Leibnitz's theory of infinitesimals. The advantages of the modern theory over that of Leibnitz are that it has precise, rigorously justified rules for manipulating infinitesimals, and that the notion of "infinitely close" can be used to represent the concept of limit, not only in connection with the reals, but in any topological context, and even in some contexts where the notion of limit is not exactly topological. For instance, a Loeb space