BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 21, Number 2, October 1989

STOCHASTIC PROCESSES AS FOURIER INTEGRALS AND DILATION OF VECTOR MEASURES

CHRISTIAN HOUDRÉ

1. Introduction. In this note we give an overview of some recent advances in representations of stochastic processes as Fourier integrals. These advances provide a Plancherel and a Hausdorff-Young theory for stochastic processes and random measures which were not previously available. We expect several applications of these methods, two of which are: existence results for linear stochastic differential equations (see Theorem 6) and a framework in which to develop a Fourier theory for the ubiquitous white noise model.

The idea of representing a stochastic process as a Fourier series or integral goes back at least to the work of Slutsky **[S]** and Cramér **[C]**, where such representations were derived for mean squared continuous stationary processes. Without the stationarity assumption, and via vector measures, Phillips **[Ph]** and Kluvánek **[K]** also obtained Fourier representation theorems for (strongly) continuous processes. Typical assumptions made to derive these Fourier integrals are the continuity of the process, the global norm boundedness of the representing vector measure, as well as its σ-additivity, or the orthogonality of its increments. However, for basic classes of stochastic processes these assumptions are not verified, e.g., a process with orthogonal increments is not necessarily continuous, while a white measure is only locally bounded. In **[Ho2]**, the above restrictions are relaxed. The presentation of the approach developed therein unfolds as follows: we first replace convergence by summability and obtain a representation theorem for the corresponding continuous processes. We then also replace continuity by measurability, and present a Plancherel and a Hausdorff-Young type theorem for random measures (Theorems 2 and 3). We then show that these random measures can be dilated to orthogonally scattered ones. For this dilation result the heart of our method is Theorem 4, which is a Grothendieck type inequality. Finally, a study of a class of linear stochastic differential equations is presented.

2. Some preliminaries. Let $(\Omega,\mathscr{B},\mathscr{P})$ be a probability space and for $1 \leq \alpha \leq 2$, let $L^\alpha(\Omega,\mathscr{B},\mathscr{P})$ ($L^\alpha(\mathscr{P})$ for short) be the space of random variables with finite αth moment (for probabilistic considerations the case $\alpha > 2$ is of little interest, furthermore included in $\alpha = 2$). On $L^\alpha(\mathscr{P})$ the norm is denoted by $\|\cdot\|_\alpha$ and for Y and Z in $L^\alpha(\mathscr{P})$, the "inner product"

Received by the editors January 31, 1989.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 60G12; Secondary 46G10.

Supported by the AFOSR Contract No. F49620 85 C 0144 and ONR No. N00014 86 C 0227.

0273-0979/89 \$1.00 + \$.25 per page