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ON SOLVABLE SUBGROUPS OF THE SYMMETRIC GROUP

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1. Introduction.. In this note we give exact values of certain invariants of the symmetric group S_n of degree n.

Let n be a positive integer, p a prime, $\sigma(G)$ the derived length and $\nu(G)$ the nilpotent length of a solvable group G. Let $\mathrm{SOLV}(n)$ denote the set of all solvable subgroups of S_n and put

$$\begin{aligned} \mathrm{SOLV}(n,p') &= \{G \in \mathrm{SOLV}(n) | p \nmid |G|\}, \\ \sigma(n) &= \max\{\sigma(G) | G \in \mathrm{SOLV}(n)\}, \\ \nu(n) &= \max\{\nu(G) | G \in \mathrm{SOLV}(n)\}. \end{aligned}$$

Similarly one defines $\sigma(n,p')$ and $\nu(n,p')$.

Let $\mathbf{N}$ be the set of all nonnegative integers. For $t \in \mathbf{N}$ we put $s(t) = \min\{m \in \mathbf{N} | \sigma(m) = t\}$ and $n(t) = \min\{m \in \mathbf{N} | \nu(m) = t\}$. For a partial ordered set L we denote by μL the set of all maximal elements in L. We put $\Sigma(t) = \{G \in \mu\,\mathrm{SOLV}(s(t)) | \sigma(G) = t\}$ and $\Sigma(t,p') = \{G \in \mu\,\mathrm{SOLV}(s(t,p'),p') | \sigma(G) = t\}$. Similarly one defines $N(t)$ and $N(t,p')$.

We define the structure of all elements of the sets $\Sigma(t)$, $\Sigma(t,p')$, $N(t)$ and $N(t,p')$.

We assume that, as permutations groups, S_m has degree m, $\mathrm{AGL}(2,3)$ has degree 9, the cyclic group $C(p)$ of order p has degree p, the groups $\mathrm{AGL}(1,p)$ and $\frac{1}{2}\,\mathrm{AGL}(1,p)$ (=the subgroup of index 2 in $\mathrm{AGL}(1,p)$) have degree p.

We say that a group W is *of type* $\{B_1^{k_1},\dots,B_s^{k_s}\}$ if W a wreath product of k_1 copies of the permutation group B_1, k_2 copies of the permutation group B_2 and so on (the order of the factors is arbitrary).

2. Main results.

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