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PROGRESS IN THE THEORY OF COMPLEX ALGEBRAIC CURVES

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ABSTRACT. We describe the contemporary view of the theory of algebraic curves over the complex numbers, with emphasis on the moduli spaces of curves and linear series on them. We then give an exposition of some of the recent work on the question of the rationality of the moduli space.

Introduction. In the last twenty years there has been a major development in our understanding of algebraic curves. A number of the classical problems have been solved and new directions of investigation have been begun. We will describe some of the history of the theory, and how it led to the modern point of view, and we will sketch proofs of many of the main assertions. Then we will explain something of how the modern ideas have been used to solve some old problems.

Many features of the current wave of progress are closely connected with patterns that go back to the earliest period in the development of the theory, so it's best to start with ancient history. To talk of complex projective curves, you need the complex numbers and you need projective space, so "ancient history" for us will start when these things become available, between about 1800 (Gauss' proof of the fundamental theorem of algebra) and 1830, (the introduction by Plücker of homogeneous coordinates for the projective plane). It goes without saying that the history below is that of a Mathematician and not of an Historian—it should probably be described as "fictionalized."

Naturally we will have to leave out parts of the theory of curves at least as rich as the parts we can put in. We beg pardon in advance from anyone whose favorite bit we've skipped. A more detailed development along the lines of this article may be found in lectures from the Bowdoin conference (Harris **[1988]**). A very beautiful survey covering a different range of topics is that of Mumford **[1976]**.

A curious aspect of the history of algebraic curves, as with other parts of algebraic geometry, is the breakdown of rigor which deeply affected the theory, leading to its virtual stagnation in a mire of unproved assertions and incomplete proofs in the end of the first third of this century. The late nineteenth and early twentieth centuries were of course a period of enormous vitality in algebraic geometry, a period in which a large fraction of our current knowledge of curves and surfaces was obtained. In some cases the low rigor was simply the result of the fact that people lacked a

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