

BOOK REVIEWS

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Class field theory, by Jürgen Neukirch, Grundlehren der Mathematischen Wissenschaften, vol. 280, Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo, 1986, \$29.50. ISBN 0-387-15251-2

Local class field theory, by Kenkichi Iwasawa. Oxford Mathematical Monographs, Oxford University Press, New York and Clarendon Press, Oxford, 1986, viii + 155 pp., \$39.95. ISBN 0-19-504030-9

Nowadays class field theory is mostly thought of as the theory which describes (more or less explicitly) the maximal abelian Galois extension of a local or global field K and which describes, again rather explicitly, the corresponding Galois group $\text{Gal}(K^{ab}/K)$ and its quotients $\text{Gal}(L/K)$, L/K abelian, preferably (and usually) in terms of some ‘norm subgroups.’ Here a local field is usually taken to be a finite algebraic extension of the field of p -adic integers \mathbf{Q}_p , or the field of Laurent series $\mathbf{F}_p((T))$ over a finite field, and a global field is a finite extension of the rationals \mathbf{Q} or of the field of rational functions $\mathbf{F}_p(T)$ over a finite field \mathbf{F}_p .

It should be noted though, that there are more general local fields over which a class field theory can be developed, in particular complete, discretely valued fields with algebraically closed residue field [9], or, more generally, with perfect residue field [3]. And, much more importantly—in my opinion—, there is the algebraic K -theory based class field theory of Kato and Parshin [6, 7, 8] for finitely generated fields over their prime field (and schemes of finite type over \mathbf{Z}). However, these last named topics are not touched upon in the two books under review so I will not say much more about them.

The “definition” of class field theory given above is quite far removed from its origin—class field theory is one of those subjects which has gone through many “revolutions,” generalizations, and changes of point of view; some 7 in my personal count and way of looking at it—and the description given does not give much of a clue to the origin of the word ‘class field.’