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CONTRIBUTIONS TO THE K-THEORY OF C^*-ALGEBRAS OF TOEPLITZ AND SINGULAR INTEGRAL OPERATORS

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Let X be a compact Hausdorff space upon which the real line $\mathbf{R}$ acts continuously, giving a transformation group or flow, and for $x \in X$ and $t \in \mathbf{R}$, let $x + t$ denote the translate of x by t. We assume throughout that the action is strictly ergodic in the sense that it is minimal and there exists a unique probability measure on X that is invariant under it. For $\varphi \in C(X)$ and $x \in X$, we write φ_x for the function on $\mathbf{R}$ defined by the formula $\varphi_x(t) = \varphi(x + t)$, $t \in \mathbf{R}$. For $x \in X$ fixed, we let $\mathfrak{S}\mathfrak{J}_x$ denote the C^*-algebra on $L^2(\mathbf{R})$ generated by the Hilbert transform and all the multiplication operators M_{φ_x} obtained by letting φ run over $C(X)$ and we let $\mathfrak{T}_x$ denote the compression of $\mathfrak{S}\mathfrak{J}_x$ to the classical Hardy space $H^2(\mathbf{R})$. These algebras arise in a number of contexts and have been objects of intensive study since the late sixties. (See the references at the end for a sampling of the literature.) In this note, we announce our results which lead to a description of the K-theory of these algebras. The algebras $\mathfrak{S}\mathfrak{J}_x$ and $\mathfrak{T}_x$ are closely related and the derivation of their K-theories involves a kind of play off between the two. In order to keep our presentation simple, we concentrate our attention on $\mathfrak{S}\mathfrak{J}_x$.

As in shown in [CMX], $\mathfrak{S}\mathfrak{J}_x$ does not depend upon x, but only on X. However, as defined, $\mathfrak{S}\mathfrak{J}_x$ is not congenial for analysis and it is helpful to represent it on different Hilbert spaces. When [CMX] was written, this was not an easy task. The following analysis remedies the situation. Let $C^*(X, \mathbf{R})$ denote the transformation group C^*-algebra associated with the flow and let $W^*(X, \mathbf{R})$ denote the double dual of $C^*(X, \mathbf{R})$. This is a huge von Neumann algebra acting on a nonseparable Hilbert space, but it is

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