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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 2, April 1989

0273-0979/89 $1.00 + $.25 per page

Hilbert modular surfaces, by Gerard van der Geer. Ergebnisse der Mathematik, Band 16, Springer-Verlag, Berlin, Heidelberg, New York, 1988, ix + 291 pp., $69.50. ISBN 3-540-17601-2

The theory of Hilbert modular surfaces is a generalisation of the classical theory of automorphic forms, and in many ways it is one of the easiest generalisations. It was started by D. Hilbert at the end of the last century, with the motivation to enhance the theory of analytic functions of several complex variables. He inspired O. Blumenthal to take up the subject, and sometimes his name is added to that of Hilbert. However, neither man got very far, and only after the general theory of complex manifolds had advanced sufficiently could progress be made in this special case. In modern times the subject has been revived by M. Rapoport and F. Hirzebruch. The theory of Hilbert modular surfaces mixes the theory of automorphic forms, arithmetic algebraic geometry (especially Shimura-varieties) and the theory of classical complex algebraic surfaces. It thus seems appropriate to give short overviews of recent developments in these subjects, and after that we try to explain how they specialize to the case of Hilbert modular surfaces. Needless to say, I tend to oversimplify the situation; for details one should consult the literature.

The theory of automorphic forms started with the classical elliptic modular functions (for a modern account see [**La**]), and has developed into a theory about reductive groups. Let us try to explain how this happened: Classically one considers the upper halfplane $\mathbf{H}$ of complex numbers with positive imaginary part, on which the group $\mathrm{SL}(2, \mathbf{R})/\{\pm 1\}$ acts by the usual $(az+b)/(cz+d)$-rule. One further chooses a subgroup Γ of finite index in $\mathrm{SL}(2, \mathbf{Z})$, and considers holomorphic functions $f(z)$ which transform under Γ according to a certain factor of automorphy, and which are holomorphic at infinity (this amounts to a certain growth-condition).