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Radically elementary probability theory, by Edward Nelson. Annals of Mathematical Studies, vol. 117, Princeton University Press, Princeton, N. J., 1987, ix + 97 pp., $40.00 ($15.00 paperback). ISBN 0-691-08473-4

The titles of few mathematics books are as bold and eye catching as this one. Does a radical proposal with perhaps illicit pleasures wait inside? Yes. As a replacement for the conventional measure theoretic foundations of probability, Nelson proposes elementary finite probability spaces and a "tiny bit of nonstandard analysis." Nonstandard analysis provides an enriched class of *finite* objects including both the *infinitely large* and the *infinitely small.* With these new *finite* objects it is possible to study a rich variety of probabilistic phenomena within the framework of finite probability spaces. These are the proposed new foundations. The book, *Radically elementary probability theory*—here after REPT—is an example of what can be done.

The book is beautifully written and essentially self-contained. A remarkably simple introduction to the necessary nonstandard analysis is presented in 13 pages. The remainder of the text develops a basic graduate course in stochastic processes. The subject matter focuses mainly on martingales and the "*Wiener walk*". Starting on p. 1 with the definition of a random variable the path goes through nonstandard analysis, martingales, the law of large numbers, the central limit theorem, the Lévy-Doob martingale characterization of the Wiener process, and the invariance principle. It is all done in 79 pp.

The treatment, although elementary, is not familiar. Theorems which look familiar are different enough to cause considerable discomfort. The uninitiated reader may be very unclear about the content, generality and flexibility of the methods and theorems. The questions of content and generality are addressed quite successfully in an appendix to which we will return later. The questions of flexibility and generality of the methods will, I think, only be settled in time but there is clearly more life in finite probability than many suspected, and there is much here that will reward careful study. The remainder of our review attempts to describe Nelson's contribution in nonstandard analysis and expands some on the changes suggested for probability.

Nonstandard analysis was created by Abraham Robinson in 1960 to lay a rigorous foundation for infinitesimals in analysis. The first step in this program was to build a new (nonstandard) enlargement (${}^*\mathbb{R}$) of the real numbers $\mathbb{R}$, which included the sought after infinitesimals. The structure of ${}^*\mathbb{R}$ was required to mirror that of $\mathbb{R}$. Two important machines, the "transfer principle" and the "standard part maps" were built to allow free movement between the standard and nonstandard worlds.

Robinson's success was spectacular. One can already see in the first edition of his book [9] which appeared in 1966, that a very powerful tool