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A formalization of set theory without variables, by Alfred Tarski and Steven Givant. Colloquium Publications, vol. 41, American Mathematical Society, Providence, R.I., 1987, xxi + 318 pp., list $60.00; ind. $36.00. ISBN 0-82189-1041-3

This last book written (in part) by Tarski is typical of his life work, in that it contributes both to our understanding of what mathematics is, as well as to technical foundational research. Very briefly, the following is done. A simple equational language $\mathscr{L}^{\times}$ is introduced, and it is shown that set theory (for example, ZFC) can be translated into equations of $\mathscr{L}^{\times}$ which have no variables, so that sentences derivable in set theory are translated into equations derivable by equational rules of inference from the translates of the set-theoretical axioms. Even more briefly, one may say that it is shown that, in principle, mathematics can be developed in the very simple framework of equations and substitution of equals for equals, rather than the customary basis in set theory formalized in first-order logic. At the very least, the main result must be considered as an impressive tour de force. It will probably influence the attitude of many mathematicians concerning the nature of their discipline. Just like the authors, this reviewer will not venture into a serious philisophical discussion of the meaning of the result.

For the exposition of the mathematics involved, very little in the way of prerequisites is needed—which is not to say that the proofs are easy. For a reader who just wants to get an idea of what is going on, without investing a lot of time in checking the proofs, the proofs can be skipped without losing the drift of the ideas. A more committed reader will find the book a rich source of ideas and problems in various foundational directions.

I found the book difficult to read. I think the reason is that formalisms are emphasized over algebraic aspects of the work. In view of the philosophical purpose of the book, this is natural; and it will probably make the book attractive to many people, especially to proof-theorists, and mathematically oriented philosophers. A model-theorist or algebraic logician might prefer to read the last chapter—applications to algebra—first.

The main ideas and results are due to Tarski, and come from the period 1940–1945. In the course of writing the book with Tarski, Givant made many independent contributions to the theory. Maddux and McNulty also contributed results or proofs to this final result of the development, and very recently Andréka and Németi solved several problems which arose during the preparation. (Since the publication, they solved the problem stated before 4.8(xvi), p. 144, positively.)

The following summary of the book, partly in different terminology from the authors', is intended as further material to help the reader decide whether to look at the book, and also to aid those who wish to study it carefully. One should also mention at the outset the useful section interdependence chart and indices.