BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 20, Number 2, April 1989

0273-0979/89 $1.00 + $.25 per page

Spectral theory and differential operators, by D. E. Edmunds and W. D. Evans. Oxford University Press, Oxford, 1987, xvi + 574 pp., $115.00. ISBN 0-19-853542-2

A linear operator is a complicated mathematical object, but many of its properties are captured in its spectrum, which is simply a set of complex numbers. For a selfadjoint linear operator, the spectrum consists of real numbers and has a quantum mechanical interpretation. If the operator represents the energy of an atomic or molecular system, the spectrum represents possible energy levels of the system. Furthermore the differences between these energy levels determine the permitted frequencies of emitted light. The rich phenomena of color are determined by the spectra of linear operators.

One striking feature of such an energy spectrum is that it usually consists of two parts. For the hydrogen atom the first part is a sequence of negative energy levels proportional to $-1/n^2$, where $n = 1, 2, 3, \ldots$, accumulating at zero. These correspond to the various bound states of the electron. The remaining part consists of all energies zero and above. These correspond to states of electrons that are free to escape.

In the general setting when H is a linear operator acting in a Banach space, the *spectrum of* H is defined as the set of complex numbers E such that $(H-E)^{-1}$ is not a bounded operator. (A *bounded* operator is an operator defined on the Banach space that maps every bounded set into a bounded set.) In particular, if E is an eigenvalue of H, then the inverse of $H - E$ does not exist, and so E belongs to the spectrum. However a number may be in the spectrum for other reasons; for instance even when $H - E$ is one-to-one it is possible that its range is not the entire Banach space.

One of the most elementary ways of dividing the spectrum into two parts is by introducing a subset called the *essential spectrum*. In the Banach space setting there are at least five inequivalent definitions of essential spectrum. Fortunately, in the case of a selfadjoint operator acting in a Hilbert space they are all equivalent. In this case the essential spectrum may be characterized as the part of the spectrum that does not consist of isolated eigenvalues of finite multiplicity. In the hydrogen atom example the essential spectrum is the interval from zero to infinity.

The reason for introducing the concept of essential spectrum is that the essential spectrum of a bounded operator is invariant under compact perturbations. (An operator is *compact* if it maps every bounded set into a set with compact closure.) This is often used in the following somewhat more general setting. Assume that

$$H = H_0 + V$$

and that V is *relatively compact* in the sense that for some fixed E not in the spectrum of H_0 the operator $V(H_0-E)^{-1}$ is compact. If both $(H-E)^{-1}$