

BOOK REVIEWS

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Fundamentals of stability theory, by John T. Baldwin. Perspectives in Mathematical Logic, Springer-Verlag, Berlin, Heidelberg, New York, 1988, xiii + 447 pp., \$60.00. ISBN 3-540-15298-9

1. The theory of classification of structures. I first met Saharon Shelah at the ICM in 1974. When I mentioned a result on universal locally finite groups which had been obtained recently by model theoretic methods, his reaction startled me: “Oh! I think I can prove a much better theorem, but tell me—what *is* a universal locally finite group?” At that time Shelah had a clear view of a very general theory of the construction and classification of algebraic systems of quite general types, based on remarkably superficial algebraic information, and less superficial model theoretic considerations. In the case of universal locally finite groups he applied this theory to justify his claim on the spot, after hearing the definition and a sketch of the earlier argument.

The problem which Shelah’s “classification theory” (also referred to as “stability theory”) is intended to solve is the following. A class K of algebraic structures is specified, for example the class of universal locally finite groups (whatever that may mean). If the isomorphism types of the structures in K can be classified in terms of intelligible numerical invariants, the theory should enable us, in principle, to find this classification. If such a classification is not possible, the theory should provide convincing evidence of this fact by enabling us to construct many structures in the class. The term “many” may mean various things, for example that a completely arbitrary structure can be set-theoretically encoded into the isomorphism type of a structure in the specified class.

Shelah calls the critical dichotomy in classification theory a “structure/nonstructure” theorem. The basic result of classification theory is that for certain reasonable classes of algebraic structures, the isomorphism types of structures in the class are either classifiable or quite wild; there is no middle ground. The paradigm for the “good” case is the class of algebraically closed fields, and analogs of transcendence degree play a role in